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On pole assignment and stabilizability of neutral type systems

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Summary. In this note we present a systematic approach to the stabilizability problem of linear infinite-dimensional dynamical systems whose infinitesimal generator has an infinite number of instable eigenvalues. We are interested in strong non-exponential stabilizability by a linear feed-back control. The study is based on our recent results on the Riesz basis property and a careful selection of the control laws which preserve this property. The investigation may be applied to wave equations and neutral type delay equations.

1 Introduction

For the linear finite-dimensional control system

$$\dot{x} = Ax + Bu, \tag{1}$$

the problem of stabilizability is naturally connected to the possibility to move, eigenvalues of A, which are in the closed right-half plane by a linear feedback $u = Fx$ to eigenvalues of $A + BF$ which are in the open left-half plane, such that the closed-loop system $\dot{x} = (A + BF)x$ becomes asymptotically stable. If we can do that with arbitrary given eigenvalues for A and $A + BF$, we say that the system is completely stabilizable (cf. [27]) or that the pole assignment problem is solvable [26]. The last property is connected with the complete controllability: the system is completely stabilizable if it is completely controllable. When the system is not completely controllable, the problem of stabilizability may be solvable if the unstable modes of the spectrum are controllable. One can then obtain asymptotic stability by feedback, which is in fact also exponential stability. It is well known that the situation in infinite dimensional spaces is much more complicated (see for example [3, 27] and references therein). The possibility to move the spectrum is not so simply connected with the controllability property and the last one also is multiply defined (exact, approximative, spectral). The spectrum itself is not sufficient to describe the asymptotic behavior of the solution of the infinite dimensional system even in Hilbert spaces [25]. Our purpose is to analyze the pole assignment problem in the particular situation of a large class of linear neutral type systems. Our approach is based one the infinite dimensional framework. Let us first give some precision about the used notions.

Let us suppose that the system (1) is given in Hilbert spaces X, for the state, and U for the control. There are essentially two notions of stabilizability : exponential and strong (asymptotic, non-exponential).

Definition 1. *The system is exponentially stabilizable if for some linear feedback F the semigroup of the closed loop system $\mathrm{e}^{(A+BF)t}$ verifies:*

$$\exists M_\omega > 1, \quad \exists \omega > 0, \quad \forall x, \quad \|\mathrm{e}^{(A+BF)t}x\| \leq M_\omega \mathrm{e}^{-\omega t}\|x\|.$$

The system is strongly stabilizable if

$$\forall x, \quad \|\mathrm{e}^{(A+BF)t}x\| \to 0, \quad \text{as} \quad t \to \infty.$$

For some particular systems the two notions are equivalent. For example it is the case for linear parabolic partial differential equations with discrete spectrum or for linear retarded systems. For linear neutral type systems and hyperbolic partial differential equations they are different: such systems may be asymptotically stable but not exponentially stable and then the same situation occurs for stabilizability (see for example [1, 18, 23, 25]). This situation is related to the location of the spectrum near the imaginary axis (see our paper about the stability problem [17]). For the neutral type systems, as for other infinite dimensional systems (for exemple hyperbolic partial differential systems), the spectrum may contain an infinite set close to the imaginary axis. In [17, 18] we gave an analysis of this situation on stability conditions. It is shown that even complete information on the location of the spectrum of the operator A (and $A+BF$) does not provide the description of the cases when the system is stable or unstable. As clearly indicated in [18, Theorem 22, p.415], there is an example of two systems (of neutral type) which have the same spectrum in the open left-half plane but one of them is asymptotically stable while the other is unstable. The reason for this is that not only the location of the spectrum (eigenvalues in the case of discrete spectrum), but also the geometric characteristics (the structure of eigenspaces and generalized eigenspaces) are important. In this connection we notice that even for particular systems (except finite-dimensional) the complete description of the stability properties is not available at the present time. In such a situation, the stabilizability problem inherits many of the technical difficulties arisen in the study of the stability.

The second main difficulty of the stabilization problem in infinite dimensional spaces is related to the action of the control, namely the dimension of the control variable and the quality of the feedback. So, it is known [24, 20, 14] that one can essentially change the spectrum of the system by use of a feedback only in the case when it is exactly controllable (this means, in particular, that the image of B is infinite dimensional). For example, it is possible to achieve for any $\mu > \mu_0$:

$$\sigma(A + BF) = \sigma(-A^* - \mu I).$$

When the control space is finite dimensional our possibilities to influence spectrum by feedback is restricted. Only a finite number of eigenvalues may be assigned by a linear feedback (bounded or A-bounded, see [11, 3]). The neutral type systems may have an infinite number of unstable eigenvalues that must be moved from the right half plane by finite dimensional feedback. In the present paper the pole assignment in this case is considered.

After a preliminary section where the infinite dimensional model of the neutral type system is given, we consider in Section 3 the case of the abstract system (1) with a one-dimensional control and with an operator A having a Riesz basis of eigenvectors. The second part of Section 3 is concerned with a more general abstract case, when there is no Riesz basis of eigenvectors. Section 4 is devoted to the case of neutral type system, after that we give some concluding remarks.

2 The neutral type system and the infinite dimensional model

We consider the following neutral type system

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)\mathrm{d}\theta \qquad (2)$$

where A_{-1} is a constant $n \times n$-matrix, $\det A_{-1} \neq 0$, A_2, A_3 are $n \times n$-matrices whose elements belong to $L_2(-1, 0)$.

In our previous work [18] we analyzed asymptotic stability conditions of the system (2). One of the main point of the cited work is the fact that for (2) it may appear asymptotic non exponential stability (see also [1] for the behavior of solutions of a class of neutral type systems). We gave a detailed analysis of non exponential stability in terms of the spectral properties of the matrix A_{-1}. As a continuation of those results we consider in the present work the control system

$$\dot{z}(t) = A_{-1}\dot{z}(t-1) + \int_{-1}^{0} A_2(\theta)\dot{z}(t+\theta)\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z(t+\theta)\mathrm{d}\theta + Bu, \quad (3)$$

where B is a $n \times p$-matrix, and study the property for this system of being asymptotic stable after a choice of a feedback control law. Namely, we say that the system (2) is asymptotically stabilizable if there exists a linear feedback control $u(t) = F(z_t(\cdot)) = F(z(t+\cdot))$ such that the system (2) becomes asymptotically stable.

It is obvious that for linear systems in finite dimensional spaces the linearity of the feedback implies that the control is bounded in every neighbourhood of the origin. For infinite dimensional spaces the situation is much more complicated. The boundedness of the feedback law $u = F(z_t(\cdot))$ depends on the topology of the state space.

When the asymptotic stabilizability is achieved by a feedback law which does not change the state space and is bounded with respect to the topology of the state space, then we call it *regular* asymptotic stabilizability. Under our assumption on the state space, namely $H^1([-1,0],\mathbb{C}^n)$, the natural linear feedback is

$$Fz(t+\cdot) = \int_{-1}^{0} F_2(\theta)\dot{z}(t+\theta)\mathrm{d}t + \int_{-1}^{0} F_3(\theta)z(t+\theta)\mathrm{d}t, \tag{4}$$

where $F_2(\cdot), F_3(\cdot) \in L_2(-1,0;\mathbb{C}^n)$.

Several authors (see for example [7, 12, 13, 4] and references therein) use feedback laws which for our system may take the form

$$\sum_{i=1}^{k} F_i\dot{z}(t-h_i) + \int_{-1}^{0} F_2(\theta)\dot{z}(t+\theta)\mathrm{d}t + \int_{-1}^{0} F_3(\theta)z(t+\theta)\mathrm{d}t. \tag{5}$$

This feedback law is not bounded in $H^1([-1,0],\mathbb{C}^n)$ and then stabilizability is not regular. If the original system is not formally stable (see [5]), i.e. the pure neutral part (when $A_2 = A_3 = 0$) is not stable, the non regular feedback (4) is necessary to stabilize. Later we shall return to this issue from an operator point of view.

Let us consider know the operator model of the system (3) used in [18] (see also [2]):

$$\dot{x} = \mathcal{A}x + \mathcal{B}u, \quad x(t) = \begin{pmatrix} y(t) \\ z_t(\cdot) \end{pmatrix}, \tag{6}$$

where $\mathcal{A}$ is the generator of a C_0-semigroup and is defined by

$$\mathcal{A}x(t) = \mathcal{A}\begin{pmatrix} y(t) \\ z_t(\cdot) \end{pmatrix} = \begin{pmatrix} \int_{-1}^{0} A_2(\theta)\dot{z}_t(\theta)\mathrm{d}\theta + \int_{-1}^{0} A_3(\theta)z_t(\theta)\mathrm{d}\theta \\ \mathrm{d}z_t(\theta)/\mathrm{d}\theta \end{pmatrix}, \tag{7}$$

with the domain

$$\mathcal{D}(\mathcal{A}) = \{(y, z(\cdot)) : z \in H^1([-1,0];\mathbb{C}^n), y = z(0) - A_{-1}z(-1)\} \subset M_2, \tag{8}$$

where $M_2 \stackrel{\mathrm{def}}{=} \mathbb{C}^n \times L_2(-1,0;\mathbb{C}^n)$. The operator $\mathcal{B} : \mathbb{C}^p \to M_2$ is defined by the $n\times p$-matrix B as follows $\mathcal{B}u \stackrel{\mathrm{def}}{=} \begin{pmatrix} Bu \\ 0 \end{pmatrix}$. The relation between the solution of the delay system (3) and the system (6) is $z_t(\theta) = z(t+\theta), \theta \in [-1,0]$.

This model was used in particular in [18] for the analysis of the stability of the system (2) and in [15] for the analysis of the controllability problems (see also [2, 10]).

From the operator point of view, the regular feedback law (4) means a perturbation of the infinitesimal generator $\mathcal{A}$ by the operator $\mathcal{BF}$ which is relatively $\mathcal{A}$-bounded (cf. [8]) and verifies $\mathcal{D}(\mathcal{A}) = \mathcal{D}(\mathcal{A} + \mathcal{BF})$. Such a perturbation does not mean, in general, that $\mathcal{A} + \mathcal{BF}$ is the infinitesimal generator of a C_0-semigroup. However, in our case, this fact is verified directly [18, 15] since after the feedback we get also a neutral type system like (2) with $\mathcal{D}(\mathcal{A}) = \mathcal{D}(\mathcal{A} + \mathcal{BF})$ (see below for more details).

From a physical point of view, $\mathcal{A}$-boundedness of the stabilizing feedback $\mathcal{F}$ means that the energy added by the feedback remains uniformly bounded in every neighbourhood of 0 (see also another point of view in [5]). Hence the problem of *regular* asymptotic stabilizability for the systems (3),(6) is to find a linear relatively $\mathcal{A}$-bounded feedback $u = \mathcal{F}x$ such that the operator $\mathcal{A}+\mathcal{BF}$ generates a C_0-semigroup $\mathrm{e}^{(\mathcal{A}+\mathcal{BF})t}$ with $\mathcal{D}(\mathcal{A} + \mathcal{BF}) = \mathcal{D}(\mathcal{A})$ and for which $\|\mathrm{e}^{(\mathcal{A}+\mathcal{BF})t}x\| \to 0$, as $t \to \infty$ for all $x \in \mathcal{D}(\mathcal{A})$.

3 Main approach to the problem of infinite pole assignment

This section presents the main methodology that we propose to solve the problem of infinite pole assignment. We consider first the case when there exists a Riesz basis of eigenvectors and the after that the case where there is only a basis of invariant subspaces.

3.1 Riesz basis of eigenvectors

This approach has been developed in [21] and then essentially extended to a more general systems as will be described in the next sections.

We consider a system

$$\dot{x} = Ax + Bu, \tag{9}$$

where A generates a contractive semigroup $\{e^{At}\}_{t\geq 0}$ in a Hilbert space H; B is a bounded operator from a Hilbert space U to H. We consider equation (9) under the assumptions:

i) A is an unbounded operator with discrete spectrum consisting of simple eigenvalues $\{\lambda_k\}_{k=1}^{\infty}$,

ii) there exists a constant $C_\sigma \equiv \frac{1}{2}\min\limits_{i\neq j}|\lambda_i - \lambda_j| > 0$, i.e. the spectrum is separated,

iii) the space U is one dimensional, so we associate B with a vector $b \in H$; besides, if $\{\phi_n\}_{n=1}^{\infty}$ is an orthonormal eigenbasis $A\phi_n = \lambda_n\phi_n$, then $b_n = \langle b, \phi_n\rangle \neq 0$, $n \in \mathbb{N}$, i.e. the system is approximatively controllable.

One of the key facts in our approach is the possibility to establish the existence of a special Riesz basis of the state space. We recall the definition.

Definition 2. *A basis $\{\psi_j\}$ of a Hilbert space H is called a Riesz basis if there are an orthonormal basis $\{\phi_j\}$ of H and a linear bounded invertible operator R, such that $R\psi_j = \phi_j$.*

If the operator A has a basis of eigenvectors, it is important also that there is a Riesz basis of eigenvectors of the operator $A + bq^*$, where by q^* we denote the functional defined by $q^*x = \langle x, q\rangle$. It is obvious that the vector q must verify some condition. It will be seen from the next sections that this key property could be extended to a more general situation (e.g. neutral type systems) where not only Riesz basis of eigenvectors, but Riesz basis of invariant subspaces should be investigated.

The following assertion is of prime importance for our considerations since it gives a simple characterization of controls which do not destroy the Riesz basis property.

Theorem 1. *Let $||b|| \cdot ||q|| < C_\sigma/2$, where $C_\sigma \equiv \frac{1}{2}\min|\lambda_i - \lambda_j| > 0$ and the family of eigenvectors $\{\phi_n\}$ of A constitute a Riesz basis of H. Then the eigenvectors ψ_k of the operator $\widetilde{A} \equiv A + bq^*$ constitute a Riesz basis of H as well.*

This result was first proved in [21] for the case of a skew-adjoint operator A. Using the Riesz basis property we have the following main result for the system (9) under the above assumptions i)-iii).

Theorem 2. *[21] Let $\{\widetilde{\lambda}_n\}_{n=1}^{\infty}$ be any set of complex numbers such that*

i) $|\lambda_n - \widetilde{\lambda}_n| < C_\sigma,\ n \in \mathbb{N}$;

ii) $\sum_{n=1}^{\infty} \frac{|\lambda_n - \widetilde{\lambda}_n|^2}{|b_n|^2} < \frac{C_\sigma}{||b||^2}$,

where $C_\sigma, b_n \equiv \langle b, \phi_n\rangle$ and λ_n are as in Theorem 1, the family of eigenvectors $\{\phi_n\}$ of A constitute a Riesz basis of H. Then there exists a unique control $u(x) = q^*x$ such that the spectrum $\sigma(\widetilde{A})$ of the operator $\widetilde{A} = A+bq^*$ is $\{\widetilde{\lambda}_n\}_{n=1}^{\infty}$ and, moreover, the corresponding eigenvectors $\widetilde{A}\psi_n = \widetilde{\lambda}_n\psi_n$, constitute a Riesz basis.

This theorem gives the description how to move the (simple) eigenvalues λ_n by using an one-dimensional bounded control (of the form $u(x) = q^*x$) inside of circles of radii proportional to $r_n \cdot b_n$, where $b_n \equiv \langle b, \phi_n\rangle$ and $\{r_n\}_{n=1}^{\infty} \in \ell^2$.

3.2 Riesz basis of invariant subspaces

In this section we present an abstract approach to the stabilization problem for a general operator model and this approach will be used in the next section to stabilize the neutral type system.

The approach presented here is a generalization of the idea proposed in [21] for the problem of infinite pole assignment for the case of the wave (partial differential) equation. In this case, the operator under consideration is skew-adjoint with a simple spectrum while in the present case it does not satisfy neither first nor second assumption. Nevertheless, the main idea of [21], after necessary improvements, allows us to treat more general case including neutral type operator model.

Here we use the notation $\mathcal{A}$ for an operator satisfying the assumptions given below. As it will be shown in the next section, the operator defined in (7) satisfies these assumptions, so the reader mainly interested in the neutral type system may simply look at $\mathcal{A}$ as at the operator (7).

We denote the points

$$\lambda_m^{(k)} = \ln|\mu_m| + \mathrm{i}(\arg\mu_m + 2\pi k), m = 1,..,\ell; k \in \mathbb{Z}$$

and the circles $L_m^{(k)}(r^{(k)})$ centered at $\lambda_m^{(k)}$ with radii $r^{(k)}$, satisfying

$$\sum_{k\in\mathbb{Z}} (r^{(k)})^2 < \infty. \tag{10}$$

Let H be a complex Hilbert space. We consider an infinitesimal generator $\mathcal{A}$ of a C_0-semigroup in H with domain $\mathcal{D}(\mathcal{A}) \subset H$. We have the following assumptions:

H1) The spectrum of $\mathcal{A}$ consists of the eigenvalues only which are located in the circles $L_m^{(k)}(r^{(k)})$, where radii $r^{(k)}$ satisfy (10). Moreover, there exists N_1 such that for any k, satisfying $|k| \geq N_1$, the total multiplicity of the eigenvalues, contained in the circles $L_m^{(k)}(r^{(k)})$, equals $p_m \in \mathbb{N}$, i.e. the multiplicity is finite and does not depend on k.

We need the spectral projectors

$$P_m^{(k)} = \frac{1}{2\pi\mathrm{i}} \int_{L_m^{(k)}} R(\mathcal{A}, \lambda)\mathrm{d}\lambda \tag{11}$$

to define the subspaces $V_m^{(k)} = P_m^{(k)} H$

H2) There exists a sequence of invariant for operator $\mathcal{A}$ finite-dimensional subspaces which constitute a Riesz basis in H. More precisely, there exists N_0 large enough, such that for any $N \geq N_0$, these subspaces are $\{V_m^{(k)}\}_{\substack{|k|\geq N\\ m=1,..,\ell}}$ and W_N, where the last one is the $2(N+1)n$-dimensional subspace spanned by all eigen- and rootvectors, corresponding to all eigenvalues of $\mathcal{A}$, which are outside of all circles $L_m^{(k)}, |k| \geq N, m = 1,..,\ell$.

The scalar product and the norm in which all the finite-dimensional subspaces $V_m^{(k)}$ and W_N are orthogonal and form a Riesz basis of subspaces are denoted by $\langle\cdot,\cdot\rangle_0$ and $\|\cdot\|_0$.

H3) The system is of single input, i.e. the operator $\mathcal{B} : \mathbb{C} \to H$ is the operator of multiplication by $\mathbf{b} \in H$.

The main result of this section is the following Theorem (the proof may be found in [19]).

Theorem 3. *[19]* **(On infinite pole assignment)**. *Assume the assumptions H1)-H3) are satisfied. Consider an infinite set of circles $L_m^{(k)}(r^{(k)})$ such that each $L_m^{(k)}(r^{(k)})$ contains only one simple eigenvalue of $\mathcal{A}$, i.e. $p_m = 1$. We denote the set of indexes of these circles by $m \in I$.*

We assume that $\mathbf{b} \in H$ is not orthogonal to eigenvectors φ_m^k, of $\mathcal{A}^*$ for $m \in I$ i.e.

$$\left\langle \mathbf{b}, \varphi_m^k \right\rangle_0 \neq 0 \quad \text{for all} \quad |k| \geq N, m \in I \tag{12}$$

and

$$\lim_{k\to\infty} k \cdot \left| \left\langle \mathbf{b}, \varphi_m^k \right\rangle_0 \right| = c_m, \in \mathbb{R} \quad \text{for all} \quad m \in I. \tag{13}$$

Then there exists $N_2 \geq N$ such that for any family of complex numbers $\widetilde{\lambda}_m^k \in L_m^{(k)}(r^{(k)})$, $m \in I, |k| \geq N_2$ there exists a linear control $\mathcal{F} : \mathcal{D}(\mathcal{A}) \to \mathbb{C}$, such that

1) the complex numbers $\widetilde{\lambda}_m^k$ are eigenvalues of the operator $\mathcal{A} + \mathcal{B}\mathcal{F}$;
2) the operator $\mathcal{B}\mathcal{F} : \mathcal{D}(\mathcal{A}) \to H$ is relatively $\mathcal{A}$-bounded.

The condition (12) means that the eigenvalues to be changed are controllable, and then may be moved. The condition (13) represents a certain boundedness of the control operator.

The result is that the controllable eigenvalues may be moved arbitrarily in some neighbourhoods of the initial eigenvalues.

4 Application to neutral type systems

The main contribution of this paper is that under some controllability conditions on the unstable poles of the system, we can assign arbitrarily the eigenvalues of the closed loop system into circles centered at the unstable eigenvalues of the operator $\mathcal{A}$ with radii r_k such that $\sum r_k^2 < \infty$. This is, in some sense, a generalization of the classical pole assignment problem in finite dimensional space. Precisely we have the following

Theorem 4. *Consider the system (3) under the following assumptions:*

1) All the eigenvalues of the matrix A_{-1} satisfy $|\mu| \leq 1$.
2) All the eigenvalues $\mu_j \in \sigma_1 \stackrel{\text{def}}{=} \sigma(A_{-1}) \cap \{z : |z| = 1\}$ are simple (we denote their index $j \in I$).

Then the system (3) is regularly asymptotic stabilizable if

3) rank $\begin{pmatrix} \Delta_{\mathcal{A}}(\lambda) & B \end{pmatrix} = n$ *for all* $\operatorname{Re}\lambda \geq 0$*, where*

$$\Delta_{\mathcal{A}}(\lambda) = -\lambda I + \lambda \mathrm{e}^{-\lambda} A_{-1} + \lambda \int_{-1}^{0} \mathrm{e}^{\lambda s} A_2(s)\mathrm{d}s + \int_{-1}^{0} \mathrm{e}^{\lambda s} A_3(s)\mathrm{d}s,$$

4) rank $\begin{pmatrix} \mu I - A_{-1} & B \end{pmatrix} = n$ *for all* $|\mu| = 1$.

In fact the controllable eigenvalues may be arbitrarily assigned in some neighbourhoods of the initial eigenvalues. Let us also precise that the neutral part of the system (namely here, the matrix A_{-1}) is not modified).

5 Conclusion

Under some controllability condition we obtain that some infinite part of the spectrum of a neutral type system may be moved arbitrarily by a finite dimensional regular feedback. The counterpart is that it may be made only in some neighbourhoods of the original eigenvalues. But for neutral type systems it is sufficient to insure asymtotic stability provided that the spectrum close to the imaginary axis is simple.

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